Sirindhorn International Institute of Technology

Thammasat University

School of Information, Computer and Communication Technology

ECS332 2016/1 Part II.3 Dr.Prapun

5 Angle Modulation: FM and PM

5.1. We mentioned in 4.2 that a sinusoidal carrier signal

$$A\cos(\underbrace{2\pi f_c t + \phi}_{\theta(t)})$$

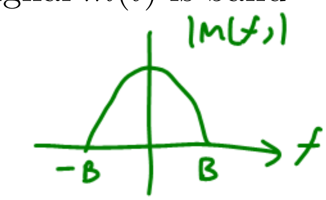
A(t) f(t) ϕ(t)

AM FM PM

We know that to adjust the freq. to f(t), it is not enough to simply change f_c to f(t) because $f(t) = \frac{1}{2\pi}\frac{d}{dt}\theta(t)$

has three basic parameters: amplitude, frequency, and phase. Varying these parameters in proportion to the baseband signal results in amplitude modulation (AM), frequency modulation (FM), and phase modulation (PM), respectively.

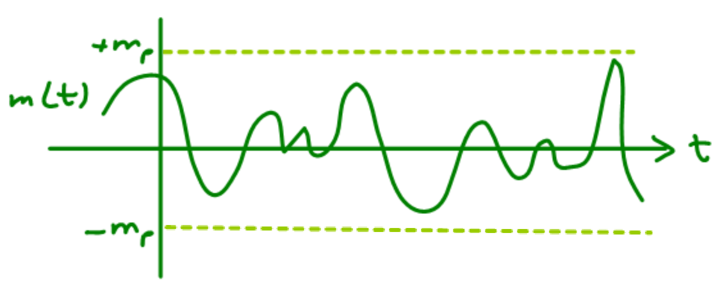
5.2. As usual, we will again assume that the baseband signal $m(t)$ is bandlimited to B; that is, ① $|M(f)| = 0$ for $|f| > B$.



As in the AM section, we will also assume that

$$② \; |m(t)| \le m_p.$$

In other words, $m(t)$ is bounded between $-m_p$ and m_p.



$-m_p \leq m(t) \leq m_p$

$-k_p m_p \leq k_p m(t) \leq \underbrace{k_p m_p}$

max. phase deviation

Definition 5.3. ***Phase modulation*** (***PM***):

$$x_{\mathrm{PM}}(t) = A\cos\left(2\pi f_c t + \phi + k_p m(t)\right)$$

Definition 5.4. The main characteristic[22] of ***frequency modulation*** (**FM**) is that the carrier frequency $f(t)$ would be varied with time so that

$$\frac{1}{2\pi}\frac{d}{dt}\theta(t) = f(t) = f_c + k_f m(t), \tag{64}$$

where k is an arbitrary constant.

- The arbitrary constant k is sometimes denoted by k_f to distinguish it from a similar constant in PM.

Example 5.5. Figure 28 illustrates the outputs of PM and FM modulators when the message is a unit-step function.

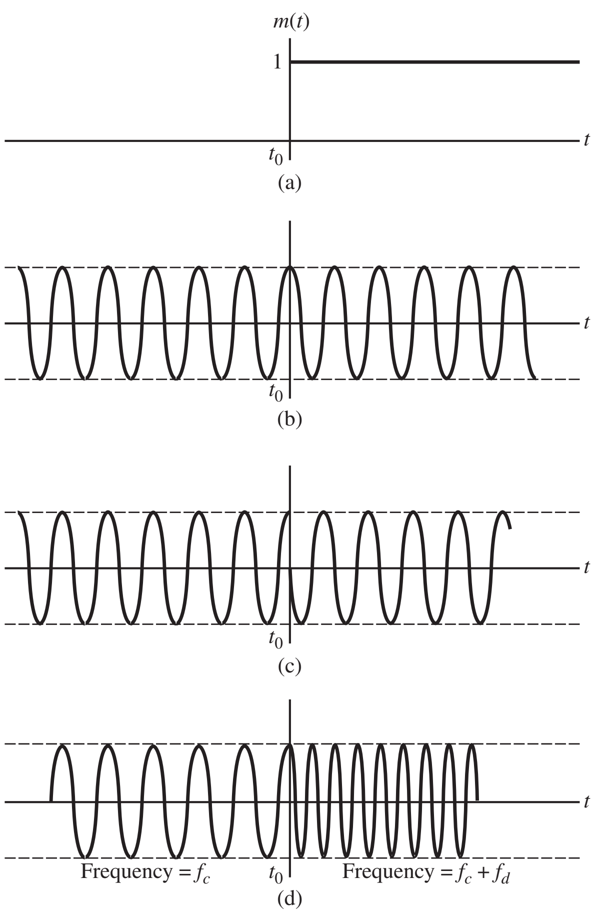


Figure 28: Comparison of PM and FM modulator outputs for a unit-step input. (a) Message signal. (b) Unmodulated carrier. (c) Phase modulator output (d) Frequency modulator output. [14, Fig 4.1 p 158]

[22] Treat this as a practical definition. The more rigorous definition will be provided in 5.15.

- For the PM modulator output,
  - the (instantaneous) frequency is f_c for both $t < t_0$ and $t > t_0$
  - the phase of the unmodulated carrier is advanced by $k_p = \frac{\pi}{2}$ radians for $t > t_0$ giving rise to a signal that is discontinuous at $t = t_0$.
- For the FM modulator output,
  - the frequency is f_x for $t < t_0$, and the frequency is $f_c + f_d$ for $t > t_0$
  - the phase is, however, continuous at $t = t_0$.

Example 5.6. With a sinusoidal message signal in Figure 29a, the frequency deviation of the FM modulator output in Figure 29d is proportional to $m(t)$. Thus, the (instantaneous) frequency of the FM modulator output is maximum when $m(t)$ is maximum and minimum when $m(t)$ is minimum.

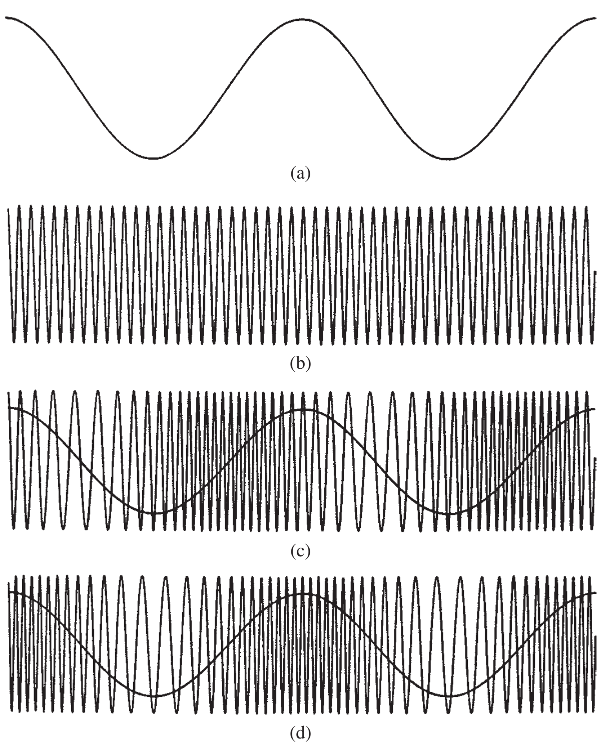


Figure 29: Different modulations of sinusoidal message signal. (a) Message signal. (b) Unmodulated carrier. (c) Output of phase modulator (d) Output of frequency modulator [14, Fig 4.2 p 159]

The phase deviation of the PM output is proportional to $m(t)$. However, because the phase is varied continuously, it is not straightforward (yet) to see how Figure 29c is related to $m(t)$. In Figure 32, we will come back to this example and re-analyze the PM output.

$m(t) = at + b$

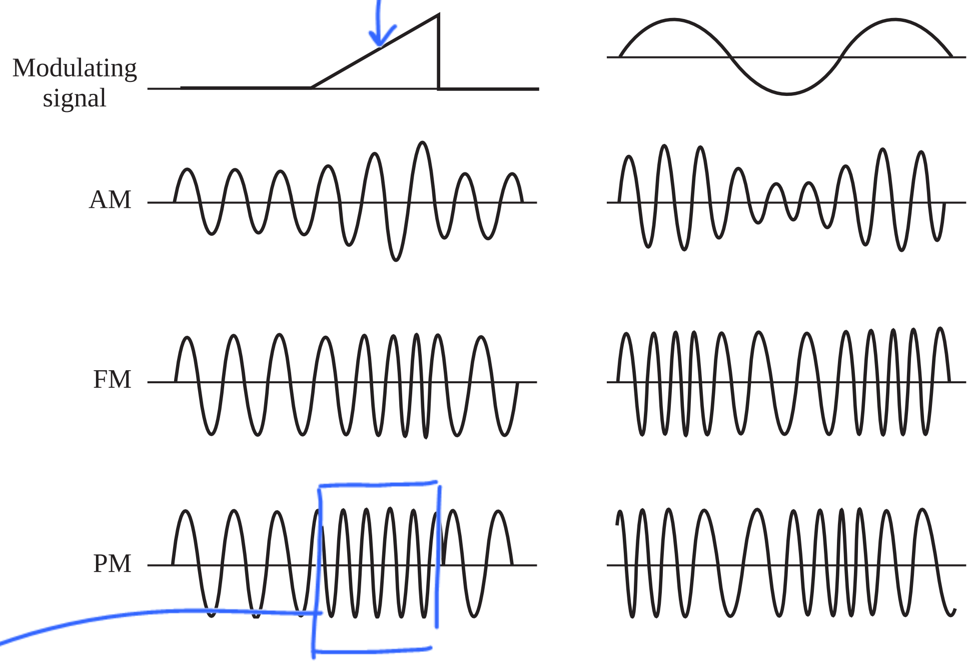


Figure 30: Illustrative AM, FM, and PM waveforms. [3, Fig 5.1-2 p 212]

Example 5.7. Figure 30 illustrates the outputs of AM, FM, and PM modulators when the message is a triangular (ramp) pulse.

To understand more about FM, we will first need to know what it actually means to vary the frequency of a sinusoid.

$$x_{PM}(t) \equiv A\cos\left(2\pi f_c t + \phi + k_p \underbrace{m(t)}_{at+b}\right) = A\cos\left(2\pi f_c t + \phi + k_p a t + k_p b\right)$$

$$= A\cos\left(2\pi\left(f_c + \frac{k_p a}{2\pi}\right)t + (\phi + k_p b)\right)$$